Exercise 82

(a) Graph the function $f(x) = x - 2\sin x$ in the viewing rectangle $[0, 8]$ by $[-2, 8]$.

(b) On which interval is the average rate of change larger: $[1, 2]$ or $[2, 3]$?

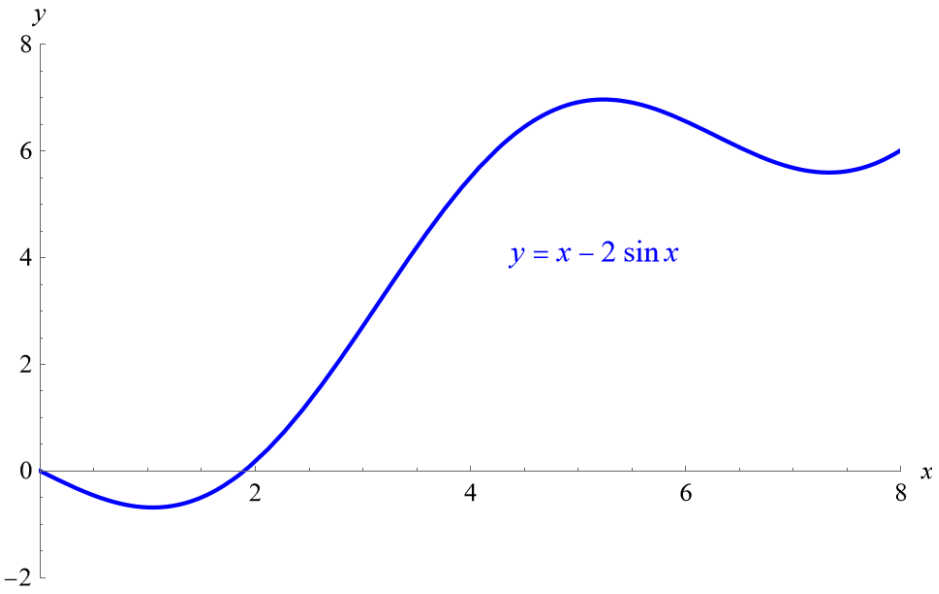
(c) At which value of x is the instantaneous rate of change larger: $x = 2$ or $x = 5$?

(d) Check your visual estimates in part (c) by computing $f'(x)$ and comparing the numerical values of $f'(2)$ and $f'(5)$.

Solution

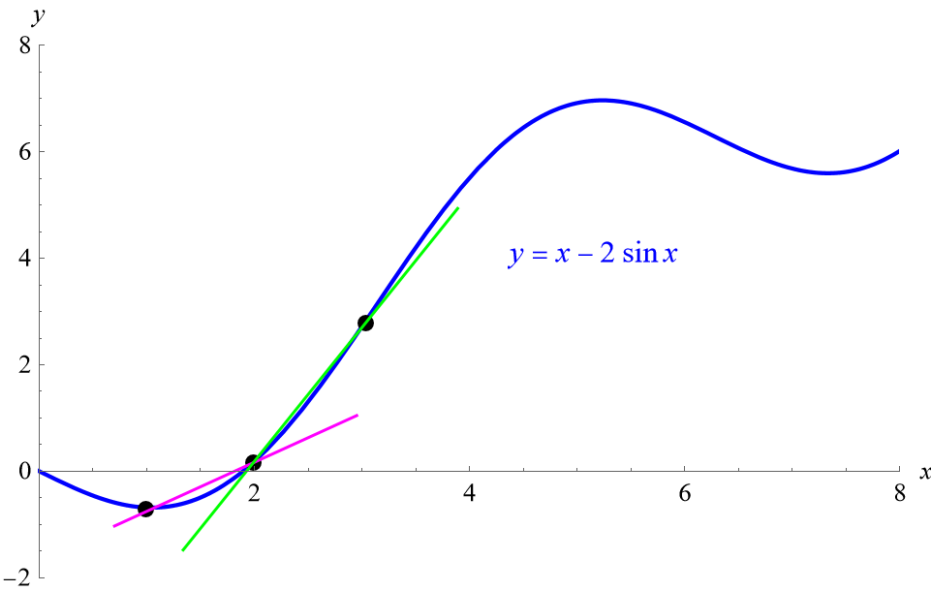
Part (a)

Below is a graph of $f(x)$ versus x on the specified viewing window.



Part (b)

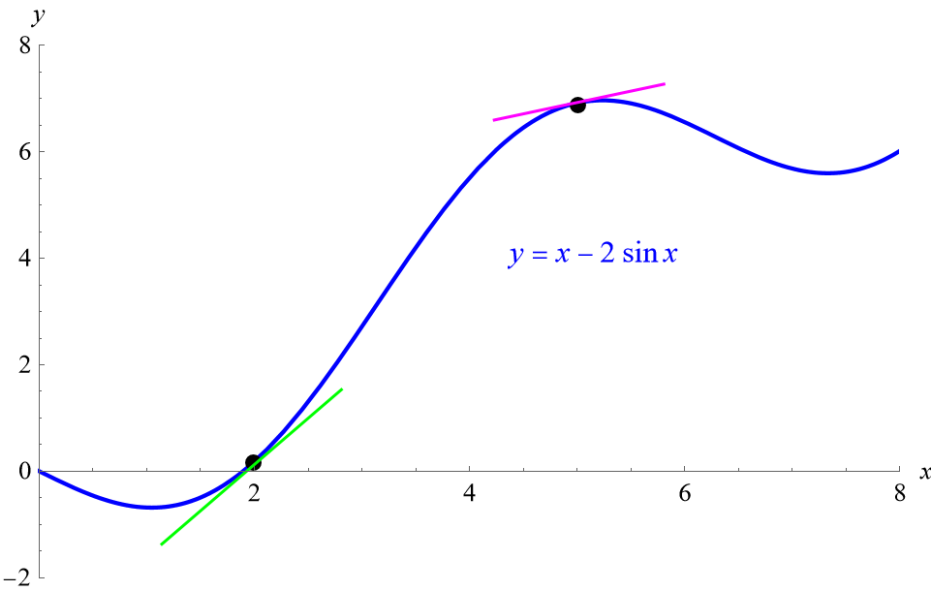
Draw secant lines through the points on the curve at $x = 1$ and $x = 2$ and $x = 2$ and $x = 3$.



Because the slope of the secant line over $[2, 3]$ is larger, the average rate of change over $[2, 3]$ is larger.

Part (c)

Draw tangent lines through the points on the curve at $x = 2$ and $x = 5$.



Because the slope of the tangent line at $x = 2$ is larger, the instantaneous rate of change at $x = 2$ is larger.

Part (d)

Take the derivative of $f(x)$.

$$f'(x) = \frac{d}{dx}(x - 2\sin x)$$

$$= \frac{d}{dx}(x) - \frac{d}{dx}(2\sin x)$$

$$= 1 - 2\cos x$$

Plug in $x = 2$ and $x = 5$.

$$f'(2) = 1 - 2\cos 2 \approx 1.83229$$

$$f'(5) = 1 - 2\cos 5 \approx 0.432676$$

Indeed, the slope of the tangent line at $x = 2$ is more than four times larger than that at $x = 5$.